Local limit theorem for structurally one-dimensional systems

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Abstract

Many important quantities can be expressed as sums over essentially one-dimensional objects connecting two distant points 0 and x,

$$g(x) = \sum_{\gamma:0\to x} w(\gamma),$$

where γ runs through "paths" connecting 0 and x, and w is some positive weight function. This is the case, for example, of the green function of random walks or self-avoiding walks, of the (finite) connectivity of percolation models, of equilibrium correlation functions of spin systems, of interfaces in two-dimensional systems, etc. The function g often has an asymptotic behaviour of the form

$$g(x) = \frac{\Psi(n_x)}{\|x\|^{(d-1)/2}}\, e^{-\xi(n_x)\|x\|}(1+o(1)),$$

as $\|x\| \to \infty$, where $n_x = x/\|x\|$, and Ψ and ξ are suitable, strictly positive, analytic functions. When this happens, the system is said to have Ornstein-Zernike behavior. In recent years, a general scheme to derive such asymptotic results non-perturbatively has emerged, and has been successfully applied to various systems of increasing complexity: SAW, subcritical Bernoulli percolation, Ising models above the critical temperature, subcritical FK-percolation (random-cluster model). I'll introduce these techniques and show how they are used in some specific cases.

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