Continuous and discontinuous solitons in polariton condensates

Stavros Komineas (University of Crete)
Stephen Shipman (Louisiana State University)
Stephanos Venakides (Duke University)

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The condensed exciton-photon system

Photons trapped in a cavity (wavefunction ψ_C) give rise to excitons (wavefunction ψ_X). The Bose condensed system is described by Schrödinger eqns with nonlinearity for mean-fields:

$$i\partial_t\psi_X = \left(\omega_X + g|\psi_X|^2\right)\psi_X + \gamma\psi_C$$
$$i\partial_t\psi_C = \left(\omega_C - \tfrac{1}{2}\partial_{xx}\right)\psi_C + \gamma\psi_X\,.$$

- The coupling constant is half the Rabi frequency $\gamma = \Omega_R/2$.
- Losses are typically included by adding imaginary part to ω_X, ω_C.
- Photon pumping would be modelled by an additional term on the rhs of the 2nd equation.

We look into the conservative system.

Static solitons

A traveling-wave in the polariton field with carrier frequency ω has the form

$$\begin{aligned}\psi_X(x,t) &= \phi_X(x-ct)e^{i(kx-\omega t)}\\ \psi_C(x,t) &= \phi_C(x-ct)e^{i(kx-\omega t)}.\end{aligned}$$

We only consider static solitons $c=0$. They satisfy (ϕ_X, ϕ_C real)

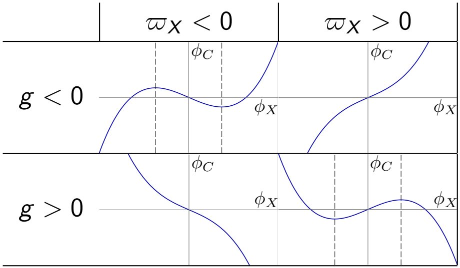
$$\begin{aligned}\left(g\phi_X^2-\varpi_X\right)\phi_X+\gamma\phi_C &= 0\\ -\tfrac{1}{2}\phi_C''-\varpi_C\phi_C+\gamma\phi_X &= 0\end{aligned}$$

with the notation $\varpi_X=\omega-\omega_X\,,\quad \varpi_C=\omega-\omega_C\,.$

Uniform solutions (far-field for solitons):

$$\begin{aligned}\phi_X^{\pm\infty} &= \pm\sqrt{\frac{1}{g}\left(\varpi_X-\frac{\gamma^2}{\varpi_C}\right)}\\ \phi_C^{\pm\infty} &= \frac{\gamma}{\varpi_C}\phi_X^{\pm\infty}\,.\end{aligned}$$

Exciton-photon relation



$$\left(g\phi_X^2 - \varpi_X\right)\phi_X + \gamma\phi_C = 0$$

A single differential equation for soliton solutions

Special case: $g\varpi_X < 0$. The 1st eqn defines a monotonic relation between ϕ_C and ϕ_X. Thus the 2nd eqn can be written in the form

$$\phi_C''(x) + U'(\phi_C(x)) = 0,$$

which results in the conservation of an energy-type function

$$\tfrac{1}{2}(\phi_C')^2 + U(\phi_C) = K \ (\mathrm{const})$$

General case

- Multiplying 1st eqn by ϕ_C' and 2nd eqn by $\gamma\phi_X'$ and adding the two integrates the system exactly.
- Eliminate ϕ_C in favor of ϕ_X to obtain a first-order ODE for $\phi_X(x)$.

1st order ODE: dark soliton solutions

Use the variable

$$\zeta(x) := g\,\phi_X(x)^2.$$

This satisfies

$$(3\zeta - \varpi_X)^2\zeta'^2 = 8\,\zeta\,Q(\zeta)\,,$$

where

$$Q(\zeta) = -\varpi_C\left[\zeta^3 - \frac{1}{2}(3\zeta_\infty + \varpi_X)\zeta^2 + \zeta_\infty\varpi_X\zeta + K\right], \quad K: \text{constant.}$$

and $\zeta_\infty = \varpi_X - \frac{\gamma^2}{\varpi_C}$.

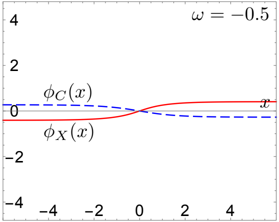
For a soliton $\zeta_S(x)$ to exist, the cubic polynomial $Q(\zeta)$ must have a double root that serves as the soliton's far-field value. Choose

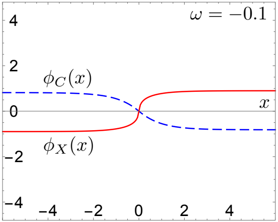
$$K = -\frac{1}{2}\frac{\gamma^2}{\varpi_C}\,\zeta_\infty^2$$

for which $Q(\zeta)$ has a double root at $\zeta = \zeta_\infty$.

Dark soliton profiles

Exciton and photon frequencies: $\omega_X = 0$, $\omega_C = 1$



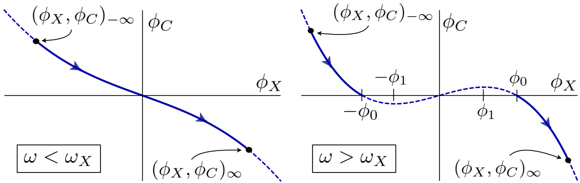


Far-field

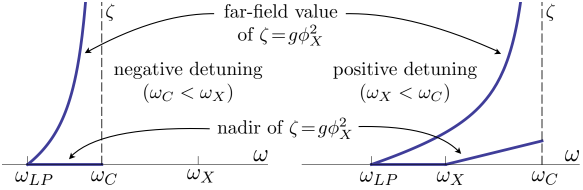
$$\zeta_\infty = \varpi_X - \frac{\gamma^2}{\varpi_C} = \omega - \omega_X - \frac{\gamma^2}{\omega - \omega_C}.$$

- $\zeta_\infty \to \infty$ for $\omega \to \omega_C$.
- $\zeta_\infty \to 0$ for $\omega \to \omega_{\mathrm{LP}}$ (lower polariton frequency).

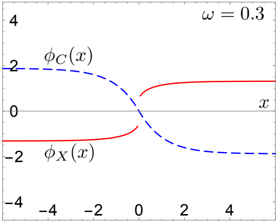
Cubic polynomial $\phi_C = \phi_C(\phi_X)$

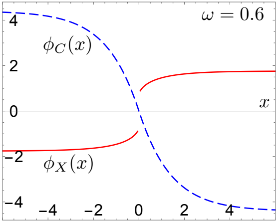


Far-field and nadir values for solitons



Dark soliton profiles





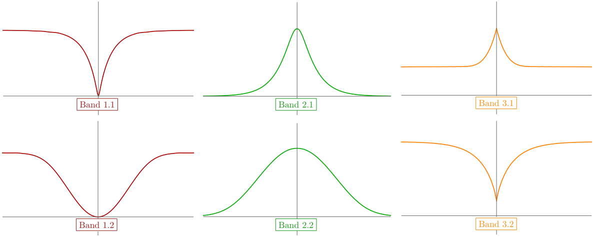
For positive detuning and $\omega_X < \omega < \omega_C$

- the exciton field jumps at $x = 0$ (center of soliton) between the values $\phi_X(0) = \pm\sqrt{\varpi_X/g}$ $(\zeta(0) = \varpi_X)$,
- the photon field is continuous and $\phi_C(0) = 0$.

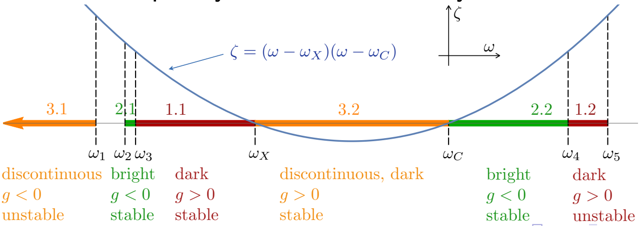
This possibility secures continuous ϕ_C and ϕ_C' at center of soliton $x = 0$, thus satisfying the differential equations.

Six bright and dark soliton bands

The square modulus $|\phi_X(x)|^2$ of the exciton field for two bright solitons, two dark solitons, two discontinuous solitons.



Six frequency bands of stationary solitons



Gross-Pitaevskii equation

- Solve the 1st equation for ϕ_X as a power series in ϕ_C up to the 3rd degree term.
- Insert $\phi_X = \phi_X(\phi_C)$ into the 2nd equation.

We obtain a GP model for the photon field

$$\frac{1}{2}\phi'' - \varepsilon\varpi_C\,\phi - \tilde{g}\phi^3 = 0.$$

where

$$\varepsilon = \frac{\gamma^2}{\varpi_X\varpi_C} - 1, \qquad \tilde{g} = (\tfrac{\varpi_C}{\varpi_X})^2 g.$$

The condition $\varepsilon = 0$ gives the polariton frequencies.

- We have a system of interacting photons (mediated by excitons).
- There seems to be no analogous way to derive a GP equation for the exciton field.

Healing length

In principle, two healing lengths ξ_X, ξ_C for exciton and photon fields:

$$\xi_X = 2\left|\frac{\phi_X(x=\pm\infty)}{\phi'_X(0)}\right| \qquad \xi_C^2 = \frac{\xi_X^2}{\eta^2}, \qquad \eta \equiv \frac{\varpi_X \varpi_C}{\gamma^2}.$$

We find

$$\xi_X^2 = \frac{4\eta^2}{\varpi_C(\eta-1)}, \qquad \xi_C^2 = \frac{4}{\varpi_C(\eta-1)}.$$

The GP model gives

$$(\xi_C^{\mathrm{GP}})^2 = \frac{4\phi(\pm\infty)^2}{\phi'(0)^2} = \frac{4\eta}{\varpi_C(\eta-1)}.$$

This underestimates the ξ_C by a factor of η. The two agree at the linear limit $\eta = 1$.

Photons in potential

Recall the coupled systems for excitons and photons. Write 1st equation as

$$\left(g\phi_X^2 - \varpi_X\right)\phi_X + \gamma\phi_C = 0 \Rightarrow \phi_X = \frac{\gamma\phi_C}{\varpi_X - g\phi_X(x)^2}\,.$$

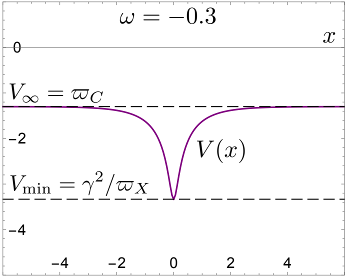
Substitute in the 2nd equation

$$-\tfrac{1}{2}\phi_C'' - \varpi_C\phi_C + \gamma\phi_X = 0 \Rightarrow -\tfrac{1}{2}\phi_C'' + V(x)\phi_C = \varpi_C\phi_C.$$

The photon field satisfies a Schrödinger eqn with potential

$$V(x) = \frac{\gamma^2}{\varpi_X - g\phi_X(x)^2}\,.$$

The photon field lives in a potential created by the exciton field.



Potential $V(x)$ due to a soliton.

Concluding remarks

- We studied the conservative exciton-photon condensed system.
- Dark and bright solitons in the photon-exciton system have been calculated, by exact integration.
- We have six soliton bands, including two bands of discontinuous solitons.
- Conservative solitons may approximately describe structures in high-Q microcavities.
- Solitons may be created in the region between two pump spots.