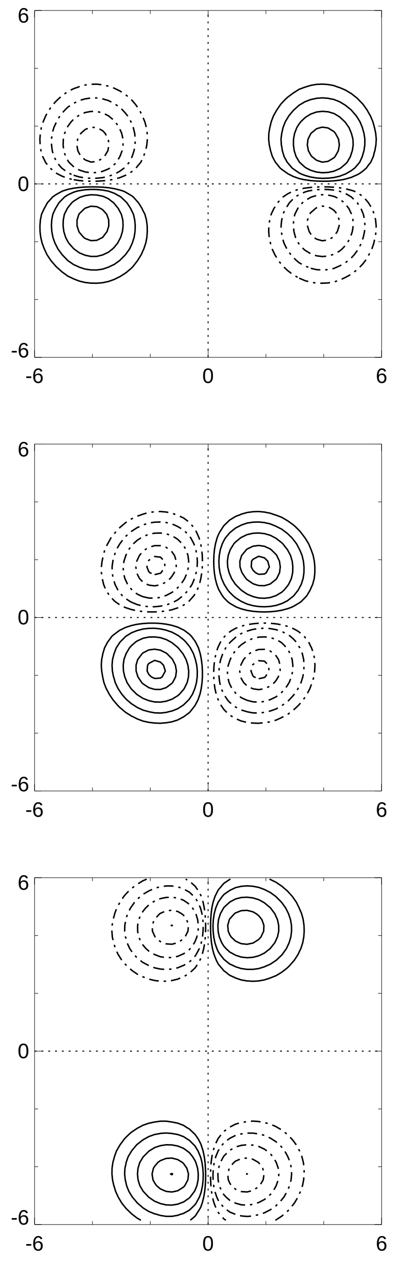
Dynamics in Magnetic Materials

Stavros Komineas

Max-Planck Institute for the
Physics of Complex Systems
Dresden



Outline

Micromagnetics

◦ The magnetization vector
◦ The Landau-Lifshitz equation

Exchange energy
Anisotropy energy
Magnetostatic energy (Maxwell's Eqns)
External fields

Exchange length
Quality factor

◦ Micromagnetic simulations

Magnetostatic field

◦ Domain walls; field-driven domain walls

Vortices and magnetic solitons

◦ Vortices: circulation, polarity, orientation

◦ Winding number

Ring particles

◦ Magnetic bubbles

Particles with perpendicular anisotropy

Magnetization dynamics

◦ Dynamics of magnetic solitons

Spontaneously pinned
Skew deflection of bubbles
Vortex-antivortex pairs

◦ Magnetic soliton dynamics in particles

Current experiments

Additionals: Antiferromagnets, Helimagnets

◦ The Heisenberg model
◦ Propagating domain walls in AFMs
◦ Spiral phase in helimagnets

Atomic magnetic moments

Atoms carry a magnetic moment $\boldsymbol{\mu}$. This is due to the motion of electrons in closed loops and it is thus associated with an atomic angular momentum $\boldsymbol{L}$:

$$\boldsymbol{\mu} = \gamma\, \boldsymbol{L},$$

where γ is a constant called the *gyromagnetic ratio*.

The energy of a moment $\boldsymbol{\mu}$ in an external magnetic field $\boldsymbol{B}$ is:

$$E = -\boldsymbol{\mu} \cdot \boldsymbol{B} = -\mu\, B\, \cos\phi,$$

where ϕ is the angle between $\boldsymbol{\mu}$ and $\boldsymbol{B}$.

This energy implies a torque $-\partial E/\partial\phi = \boldsymbol{\mu} \times \boldsymbol{B}$. The equation of motio is:

$$\frac{d\boldsymbol{\mu}}{dt} = \gamma\, \boldsymbol{\mu} \times \boldsymbol{B}.$$

Example: Consider $\boldsymbol{B} = (0, 0, B)$. Find the solution for $\boldsymbol{\mu}$ precessing around $\boldsymbol{B}$.

Magnetization

We define the *magnetization* $\boldsymbol{M}$ as the magnetic dipole moment per unit volume:

$$\boldsymbol{M} = \frac{1}{V} \sum \boldsymbol{\mu}_i \approx \frac{1}{V} \int_V \boldsymbol{\mu} \, dV.$$

By analogy to the atomic dipole moment $\boldsymbol{\mu}$, we suppose that $\boldsymbol{M}$ is a vector of constant length:

$$|\boldsymbol{M}| = M_s, \qquad M_s : \textit{saturation magnetization}.$$

The vector $\boldsymbol{M} = (M_x, M_y, M_z)$ may vary in space and time: $\boldsymbol{M} = \boldsymbol{M}(x, y, z, t)$. It can also be expressed in terms of two angles $0 \le \Theta \le \pi$, $0 \le \Phi < 2\pi$:

$$\begin{aligned} M_x &= M_s \cos\Phi \, \sin\Theta, \\ M_y &= M_s \sin\Phi \, \sin\Theta, \\ M_z &= M_s \cos\Theta, \end{aligned}$$

The magnetic energy is $E = -\int \boldsymbol{M} \cdot \boldsymbol{B} \, dV$.

Energy in a Ferromagnet

Exchange energy

$$E_{\mathrm{ex}} \sim -\boldsymbol{M}_\alpha \cdot \boldsymbol{M}_\beta \longrightarrow -\boldsymbol{M}_i \cdot (\boldsymbol{M}_{i+1} + \boldsymbol{M}_{i-1}) \longrightarrow \boldsymbol{M}\,\frac{d^2\boldsymbol{M}}{dx^2}.$$

In the three-dimensional space we write

$$\begin{aligned} E_{\mathrm{ex}} &= -\frac{A}{M_s^2}\int \boldsymbol{M}\cdot\boldsymbol{\nabla}^2\boldsymbol{M}\,dV = \frac{A}{M_s^2}\int \partial_i\boldsymbol{M}\cdot\partial_i\boldsymbol{M}\,dV = \qquad (i=1,2,3) \\ &= \int (\partial_x\boldsymbol{M}\cdot\partial_x\boldsymbol{M} + \partial_y\boldsymbol{M}\cdot\partial_y\boldsymbol{M} + \partial_z\boldsymbol{M}\cdot\partial_z\boldsymbol{M})\,dV. \end{aligned}$$

A is called the *exchange constant* (typically $A \sim 10^{-11}\mathrm{J/m}$).

The exchange energy is minimum as long as the spins are aligned (uniform magnetization: $\boldsymbol{M}(x,y,z) = \mathrm{constant\ vector}$).

Anisotropy energy

Gives rise to a prefered direction for the magnetization. Generally the anisotropy term has the same symmetry as the crystal structure of the material and we call it a *magnetocrystalline anisotropy*.

The simplest case is a uniaxial anisotropy (K: anisotropy constant):

$$E_a = -\frac{K}{M_s^2} \int (M_z)^2 \, dV \rightarrow \frac{K}{M_s^2} \int (M_x^2 + M_y^2) \, dV.$$

This is an *on-site* term which favours: $\boldsymbol{M} = \pm M_s \hat{\boldsymbol{z}}$ ("plus" and "minus" are equally favoured) $\longrightarrow$ The z is called the easy axis.

E.g., hexagonal cobalt exhibits uniaxial anisotropy: $K = 4.5 \times 10^5 \mathrm{J/m}^3$.

We can also have easy-plane anisotropy:

$$E_a = \frac{K}{M_s^2} \int (M_z)^2 \, dV.$$

We also, have the case of cubic anisotropy for cubic crystals such as iron and nickel.

The magnetostatic field and energy

Magnetic moments give rise to a magnetic field and they thus interact with neighbouring magnetic moments (dipole-dipole interactions).

The magnetic field H of a magnet satisfies Maxwell's equations in matter (assume the fields are time-independent):

$$\nabla \times \boldsymbol{H} = 0, \qquad \nabla \cdot \boldsymbol{B} = 0, \quad (\boldsymbol{B} \equiv \boldsymbol{H} + 4\pi \boldsymbol{M}).$$

H: the magnetostatic field.

The magnetostatic energy is

$$E_m = -\frac{1}{2} \int \boldsymbol{M} \cdot \boldsymbol{H} \, dV.$$

Total energy

Finally, the total energy can be written in the form:

$$E = E_{\mathrm{ex}} + E_a + E_m = -\frac{1}{2} \int \boldsymbol{M} \cdot \left[\frac{2A}{M_s^2} \boldsymbol{\nabla}^2 \boldsymbol{M} + \frac{2K}{M_s^2} M_z \, \hat{\boldsymbol{z}} + \boldsymbol{H} \right] dV.$$

This indicates that the magnetization feels an effective field (add a possible external field $\boldsymbol{H}_{\mathrm{ext}}$):

$$\boldsymbol{F}_{\mathrm{eff}} \equiv \frac{2A}{M_s^2} \boldsymbol{\nabla}^2 \boldsymbol{M} + \frac{2K}{M_s^2} M_z \, \hat{\boldsymbol{z}} + \boldsymbol{H} + \boldsymbol{H}_{\mathrm{ext}}.$$

The Landau-Lifshitz equation

The dynamics of the magnetization is described by the Landau-Lifshitz equation:

$$\frac{\partial \boldsymbol{M}}{\partial t} = -\boldsymbol{M} \times \left[\frac{2A}{M_s^2} \Delta \boldsymbol{M} + \frac{2K}{M_s^2} M_z \, \hat{\boldsymbol{z}} + \boldsymbol{H} + \boldsymbol{H}_{\mathrm{ext}} \right].$$

The Landau-Lifshitz equation in simpler form

Introduce new units:

Unit of length: $\ell_{\mathrm{ex}} \equiv \sqrt{A/(2\pi M_s^2)}$ (exchange length).

Unit of time: $\tau \equiv 1/\sqrt{(4\pi M_s^2\,\gamma)}$.

Normalize the fields (so that $\boldsymbol{m}^2 = 1$): $\boldsymbol{m} \equiv \dfrac{M}{M_s}, \quad \boldsymbol{h} \equiv \dfrac{\boldsymbol{H}}{4\pi M_s}, \quad \boldsymbol{h}_{\mathrm{ext}} \equiv \dfrac{\boldsymbol{H}_{\mathrm{ext}}}{4\pi M_s}$.

With these substitutions the Landau-Lifshitz equation becomes:

$$\frac{\partial \boldsymbol{m}}{\partial t} = -\boldsymbol{m} \times \boldsymbol{f}, \quad \boldsymbol{f} = \Delta \boldsymbol{m} + Q\, m_3\, \hat{\boldsymbol{z}} + \boldsymbol{h} + \boldsymbol{h}_{\mathrm{ext}}.$$

We have defined the important quantity: $Q \equiv \dfrac{K}{2\pi M_s^2}$ (quality factor).

The energy is now:

$$E = \frac{1}{2}\int \partial_i \boldsymbol{m} \cdot \partial_i \boldsymbol{m}\, dV + \frac{Q}{2}\int (m_3)^2\, dV - \frac{1}{2}\int \boldsymbol{h} \cdot \boldsymbol{m}\, dV - \int \boldsymbol{h}_{\mathrm{ext}} \cdot \boldsymbol{m}\, dV$$

Magnetic domain walls

Consider a bulk ferromagnet which is magnetized "up" ($\boldsymbol{M} = M_s \hat{\boldsymbol{z}}$) on one end, and "down" ($\boldsymbol{M} = -M_s \hat{\boldsymbol{z}}$) on its other end. A domain wall exists between the two domains.

Landau and Lifshitz (1935) have given the form of this wall:

$$m_z = \tanh(x\sqrt{Q}) \Longrightarrow M_z = M_s \tanh(x/\sqrt{K/A}),$$

$$m_y = 1/\cosh(x\sqrt{Q}) \Longrightarrow M_y = M_s/\cosh(x/\sqrt{K/A}).$$

This solution satisfies the Landau-Lifshitz equation.

The domain wall width is $\delta = \sqrt{K/A}$.

It is called a Bloch wall (find the magnetostatic field of a Bloch wall!).