Existence of curved travelling front solution of a model of spreading depression

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Spreading Depressions (SD) are transient suppressions of all neuronal activities that spread through the gray matter of the brain during a stroke or a migraine with aura for example. SD are observed and well known in the rodent brain, but their existence is very discussed in the human brain.

In biological models, SD are created by a reaction-diffusion mechanism in the gray matter and are progressively absorbed in the white matter. Here we will thus have interest in the existence of a curved travelling front $u(t,x,y) = U(x-ct,y)$ solution of the following equation.

$$\partial_t u - \triangle u = f(u)1_{\{|y|\leq R\}} - \alpha u 1_{\{|y|>R\}}$$

where $f(u) = \lambda u(u-\theta)(1-u)$ and $X = (x,y) \in \mathbb{R}^N$ with $x \in \mathbb{R}$ and $y \in \mathbb{R}^{N-1}$.

We will prove that there exists $R_0 > 0$ such that for $R < R_0$ no travelling front is solution of this equation by studying the possible asymptotic profiles. Then we will prove the existence of a front for $R > R_1 \geq R_0$ using the method developped by Risler. The idea is to study in detail the energy of a solution of the equation with an initial condition in various travelling referential in order to prove the "convergence" toward a travelling front.