Lectures on differential equations of quantum physics

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Differential equations of quantum physics is a relatively young and fast developing field of mathematics and mathematical physics. Its origin is in the last fundamental equation of physics - the Schrödinger equation, describing quantum matter such as atoms, molecules, solids and ... stars.

Soon after the Schrödinger equation was written it was realized that it is intractable beyond two particle systems and a search for effective approximations began. This resulted in the key effective quantum equations

- Hartree and Hartree-Fock equations
- Gross-Pitaevski (or nonlinear Schrödinger) equation
- Kohn-Sham equation (density functional theory)
- Ginzburg-Landau equations

These equations trade the large number of degrees of freedom (=3 times the number of particles) for nonlinearity and treat quantum particle systems as continuous media.

In this talk, I will introduce the equations above as well as the Chern-Simons equations (and possibly the Yang-Mills equations), describe their origins, physical (and often geometrical) significance, properties and applications. I will also review some recent results.

The lectures will use only basic results from analysis and geometry and otherwise will be self-contained.